QUIZ 1. Propositions

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QUIZ 1. VARIANT 1

1. Prove by using laws of logical equivalence that
   (a) $\neg(p \to q) \wedge q \sim false$;
   (b) $p \vee \neg(q \wedge p) \sim true$;
   (c) $p \wedge [(p \vee r) \wedge (q \vee r)] \sim (p \wedge q) \vee (p \wedge r)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from the 3 floor to 2 and 1 floors.
   (b) Give a definition of the disjunctive normal form.

3. Determine without using truth tables whether the following propositions are tautologies
   (a) $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$,
   (b) $(p \to q) \to [(\neg p \to q) \to \neg q]$,
   (c) $[\neg p \wedge (p \to q)] \to \neg q$.

4. Find a proposition which is equivalent to $\neg[(\neg p \to q) \to \neg p] \vee (p \to (q \wedge r))$ and involves only the following logical operator(s):
   (a) $\{\wedge, \neg\}$,
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

$$\Phi(p, q, r) \text{ is false iff } (p, q, r) \in \{(0, 0, 0), (1, 1, 1), (0, 1, 0), (0, 0, 1)\}.$$

6. Construct a truth table for $p \oplus ((q \mid p) \wedge (p \leftrightarrow r))$.

Group	1	2	3	4	5	6	Total
	| |	|	| |	|			

QUIZ 1. VARIANT 2

1. Prove by using laws of logical equivalence that
   (a) $\neg(p \vee q) \wedge q \sim false$;
   (b) $\neg p \vee \neg(q \wedge \neg p) \sim true$;
   (c) $(p \vee q) \wedge (p \vee r) \wedge (q \vee s) \wedge (r \vee s) \sim (p \wedge s) \vee (q \wedge r)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from 4 floor to 3 and 1 floors.
   (b) Give a definition of functionally complete system.

3. Determine without using truth tables whether
   (a) $(\neg p \to \neg q) \to [(\neg p \to \neg q) \to p]$
   (b) $[p \to (q \to r)] \to [(p \wedge q) \to r]$
   (c) $p \vee (q \wedge \neg p)$
are tautologies.

4. Find a proposition which is equivalent to $\neg[(\neg p \wedge q) \to \neg p] \wedge \neg[(p \wedge q) \to \neg q]$ and involves only the following logical operator(s):
   (a) $\{\vee, \neg\}$,
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

$$\Phi(p, q, r) \text{ is true iff } (p, q, r) \in \{(1, 0, 0), (0, 1, 1), (0, 0, 0), (1, 0, 1)\}.$$

6. Construct a truth table for $(r \oplus p) \downarrow ((p \leftrightarrow q) \to r)$.

Group	1	2	3	4	5	6	Total
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QUIZ 1. VARIANT 3

1. Prove by using laws of logical equivalence that
   (a) $\neg(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q) \sim false$;
   (b) $(\neg(q \wedge p) \wedge \neg p) \vee p \sim true$;
   (c) $(p \vee q) \wedge (q \vee r) \wedge (r \vee s) \sim (p \wedge r) \vee (q \wedge r) \vee (q \wedge s)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from 5 floor to 3 and 2 floors.
   (b) Give a definition of the conjunctive normal form.

3. Determine without using truth tables whether
   (a) $p \to (q \to p)$,
   (b) $[(p \to q) \wedge (q \to r)] \to [p \to (q \wedge r)]$,
   (c) $[\neg p \wedge (p \vee q)] \to q$
are tautologies.

4. Find a proposition which is equivalent to $[\neg(\neg p \wedge r) \vee \neg s] \wedge [(q \wedge \neg s) \to r]$ and involves only the following logical operator(s):
   (a) $\{\to, \neg\}$,
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

   $\Phi(p, q, r)$ is true iff the most of values of elementary propositions are true.

6. Construct a truth table for $(r \vee q) \oplus ((p \leftrightarrow q) \downarrow r)$.

Group	1	2	3	4	5	6	Total
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QUIZ 1. VARIANT 4

1. Prove by using laws of logical equivalence that
   (a) $q \wedge \neg(\neg p \vee q) \sim false$;
   (b) $\neg p \to \neg(q \wedge p) \sim true$;
   (c) $p \wedge (p \vee r) \wedge (q \vee r) \sim (p \wedge q) \vee (p \wedge r)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from 5 floor to 4 and 2 floors.
   (b) Give a definition of a tautology and of a contradiction.

3. Determine without using truth tables whether
   (a) $(\neg p \to \neg q) \to (q \to p)$,
   (b) $[p \to (q \vee \neg r)] \to [(p \wedge q) \to (p \to r)]$,
   (c) $p \to (\neg q \to p)$
are tautologies.

4. Find a proposition which is equivalent to $[\neg p \wedge (r \to \neg q)] \vee (r \to s)$ and involves only the following logical operator(s):
   (a) $\{\wedge, \neg\}$
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

   $\Phi(p, q, r)$ is false iff even numbers of values of elementary propositions are true.

6. Construct a truth table for $(p \mid q) \leftrightarrow ((p \vee q) \downarrow r)$.

Group	1	2	3	4	5	6	Total
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QUIZ 1. VARIANT 5

1. Prove by using laws of logical equivalence that
   (a) $\neg(\neg(p \vee (p \wedge q)) \vee p) \sim false$;
   (b) $p \to (q \to p) \sim true$;
   (c) $(p \vee q) \wedge (q \vee r) \wedge (r \vee s) \sim (p \wedge r) \vee (q \wedge r) \vee (q \wedge s)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from from 4 floor to 3 and 1 floors.
   (b) Give a definition of a dual proposition.

3. Determine without using truth tables whether
   (a) $\neg p \to [\neg q \to \neg(p \to q)]$,
   (b) $[\neg q \wedge (p \to q)] \to \neg p$,
   (c) $p \to [(q \wedge p) \to \neg p]$
are tautologies.

4. Find a proposition which is equivalent to $[p \to (\neg q \to r)] \wedge [(p \wedge q) \vee r]$ and involves only the following logical operator(s):
   (a) $\{\to, \neg\}$,
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

   $\Phi(p, q, r)$ is true iff exactly two values of elementary propositions are false.

6. Construct a truth table for $(q \downarrow r) \wedge ((r \downarrow p) \downarrow r)$.

Group	1	2	3	4	5	6	Total
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QUIZ 1. VARIANT 6

1. Prove by using laws of logical equivalence that
   (a) $(p \wedge q) \wedge \neg(q \to p) \sim false$;
   (b) $(p \wedge q) \to q \sim true$;
   (c) $(p \vee q) \wedge (p \vee r) \wedge (q \vee s) \wedge (r \vee s) \sim (p \wedge s) \vee (q \wedge r)$.

2. (a) Construct a contact schema managing the electric motor of the lift descending from 5 floor to 3 and 2 floors.
   (b) Give a definition of a proposition.

3. Determine without using truth tables whether
   (a) $[\neg p \wedge (p \vee q)] \to q$,
   (b) $[p \to (q \vee r)] \to [(p \to q) \to (p \to r)]$,
   (c) $(p \vee q) \to \neg(p \wedge q)$
are tautologies.

4. Find a proposition which is equivalent to $[p \to \neg q] \to [(\neg p \wedge q) \vee \neg q]$ and involves only the following logical operator(s):
   (a) $\{\vee, \neg\}$,
   (b) either $\{|\}$ or $\{\downarrow\}$ (up to your choice).

5. Construct the formula $\Phi(p, q, r)$ such that

   $\Phi(p, q, r)$ is true iff all elementary propositions have the same value.

6. Construct a truth table for $(p \mid r) \oplus ((r \mid p) \to r)$.

Group	1	2	3	4	5	6	Total
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